

A Comment on a Recent Derivation of the Born Rule by Zurek

Cecilia Jarlskog

Division of Mathematical Physics, Physics Department
LTH, Lund University, Lund, Sweden

Abstract

The derivation of the Born rule by Zurek uses a “splitting procedure” where a physical state is subdivided into a number of states. It is argued that in quantum field theory, which encompasses quantum mechanics, such a procedure would in general modify the physics.

Understanding the origin of Born rule [1] in quantum mechanics is one of the greatest challenges in physics. This rule is also valid in quantum field theory which encompasses quantum mechanics. Briefly, suppose that we have a superposition of the form

$$|\Psi\rangle = \sum_{i=1}^n c_i |\psi_i\rangle, \quad (1)$$

where the c_i 's are constants and the state vectors $|\psi_i\rangle$ form a complete orthonormal system. Moreover, $\sum_{i=1}^n |c_i|^2 = 1$, to ensure that Ψ is properly normalized. Simply stated, the Born rule tells us that $|c_i|^2$ gives the probability of finding $|\Psi\rangle$ in the state $|\psi_i\rangle$. By now, 85 years after its inception, no violation of this rule has been found.

During the past several years Zurek has made great efforts to derive Born rule by utilizing other postulates of quantum mechanics. According to his own account, his work has been inspired by progress made in the domain of decoherence in quantum mechanics (see Ref. [2] for an excellent pedagogical introduction to this subject). In a recent paper [3] Zurek gives an update of his derivation. He begins by considering a state of the form (which in this note we denote by $|F\rangle$)

$$|F\rangle = |s_1\rangle + |e_1\rangle + |s_2\rangle + |e_2\rangle, \quad (2)$$

where $|s_j\rangle$ and $|e_j\rangle$, $j=1,2$, may be thought of as the two states of a system S and its "environment" E . We note in passing that $|F\rangle$ is very special as it can be rewritten in the form

$$|F\rangle = (|s_1\rangle, |s_2\rangle) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} |e_1\rangle \\ |e_2\rangle \end{pmatrix}. \quad (3)$$

$|F\rangle$ may be looked upon as the scalar product of the two vectors (dropping for the moment the ket symbols) $\vec{s} = (s_1, s_2)^T$ and $\vec{e} = (e_1, e_2)^T$. Thus $F (= \vec{s}^T \vec{e})$ is a highly symmetric state. Indeed, it is invariant under rotations $R(\theta)$

$$\vec{s} \rightarrow R(\theta) \vec{s}, \quad \vec{e} \rightarrow R(\theta) \vec{e},$$

the angle θ being arbitrary. This means that all these rotated states of the system are equivalent and appear entangled with their corresponding environmental partners with equal coefficients.

Returning to Zurek's work, his next step is to replace the states $|e_j\rangle$ by a sum over a larger number of states

$$|e_1\rangle = \frac{1}{\sqrt{n_1}} \sum_{l=1}^{n_1} |e'_l\rangle, \quad |e_2\rangle = \frac{1}{\sqrt{n_2}} \sum_{l=1}^{n_2} |e''_l\rangle, \quad (4)$$

where n_1 and n_2 are integers that could be very large and

$$\langle e'_k | e'_j \rangle = \langle e''_k | e''_j \rangle = \delta_{jk}, \quad \langle e'_k | e''_j \rangle = 0.$$

All in all, Zurek cuts a single environmental state into a number of other states, keeping the norms properly intact. This procedure looks innocuous, but is it?

The point I wish to make is that in quantum field theory splitting a state into substates is expected to lead to new physics. Thereby, the modified version is not necessarily the same as the original theory for which the proof was to be given.

As an example consider the state of a neutral pion in the quark model. In the original quark model, the neutral pion was described by the superposition

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle, \quad (5)$$

where u and d denote the up and down quarks respectively and the bar stands for the antiparticle. With the discovery that the quarks are “colored”, i.e., each of them comes in three equivalent varieties, one had to replace $u\bar{u}$ by $\frac{1}{\sqrt{3}}\sum_{j=1}^3 u^j\bar{u}^j$ and perform a similar replacement for the down quark. Here j stands for the color degree of freedom. [This construction is similar to what Zurek does in his proof.] Thus the neutral pion state becomes

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\sum_{j=1}^3|u^j\bar{u}^j - d^j\bar{d}^j\rangle. \quad (6)$$

Note that the $\sqrt{3}$ ensures that the pion state is properly normalized. Nonetheless, the fact that the “colorless” (or one-color, if you prefer) state in equation (5) has been replaced with three leads to different physics, in spite of the fact that color is confined, i.e., it does not leak out. For example, the above modification changes the prediction of the neutral pion lifetime by a factor of nine!

An interesting example, where a state is indeed replaced by a superposition of other states, is found in the Pauli-Villars regularization scheme [4] in quantum electrodynamics. In this case the field of the electron is replaced by a sum which includes a number of other hypothetical fields, with the same quantum numbers as the electron. However, these auxiliary fields are taken to be very massive (so that they can not be produced) and appear in the theory with *negative probabilities*. In this manner, one is able to keep the physics intact up to a large energy scale, beyond which the theory hits the domain of negative probabilities and becomes unphysical.

The upshot of this note is that splitting a state into substates leads to new physics in quantum field theory (which encompasses quantum mechanics). The ensuing

theory is not, in general, equivalent to the original one. Therefore, a proof based on the above cutting procedure is not necessarily conclusive.

References

- [1] M. Born, Z. Physik 37 (1926) 863; 38 (1926) 803
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- [3] W. H. Zurek, Phys. Rev. Lett. 106 (2011) 250402. This article contains references to Zurek’s earlier work.
- [4] W. Pauli and F. Villars, Rev. Mod. Phys. 21 (1949) 434